SOME THEORETICAL PROBLEMS OF DEEP INELASTIC LEPTON-HADRON INTERACTIONS

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The interesting results obtained by the SLAC—MIT group [1] on the deep inelastic scattering of electrons on protons, a similar neutrino experiment at CERN [2] as well as the Columbia — BNL group experiment [3] on the massive muon pair production in hadron collisions have stimulated much theoretical work. The present report is a review of recent works performed in Dubna by V. A. Matveev, A. N. Tavkhelidze, R. F. Kögerler and myself concerning the study of deep inelastic lepton-hadron processes on the basis of

I) automodelity or scale invariance principle,
II) current algebra,
III) vector dominance.

1. Automodelity

The SLAC experiments on deep inelastic electron-proton scattering have shown that the cross section above the resonance excitation region is large (comparable with the «point» Mott cross section) and decreases with increasing momentum transfer much slower than the cross section for elastic scattering or production of a resonance. The attention to such a possibility had been drawn by M. A. Markov [4] before appropriate experimental information was available.

The process of inelastic scattering of electrons on a nucleon

$$e^- + p \to e^- + p' + \cdots \tag{1}$$

is described by the diagram of Fig. 1.

If a single electron is detected in the final state then the cross section is expressed in terms of two structure functions W_1 (ν, q^2) and W_2 (ν, q^2) which depend upon two Lorentz-invariant variables: $\nu = pq = m(E - E')$ — the energy transfer from electron to hadrons or the virtual photon energy in the lab. system, $q^2 = -4EE' \sin^2 \vartheta/2$ — the squared four-momentum transfer or the virtual photon mass, in the follwing manner

$$\frac{d\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left(mW_2 + \frac{2}{m} \operatorname{tg}^2 \vartheta/_2 W_1\right). \tag{2}$$

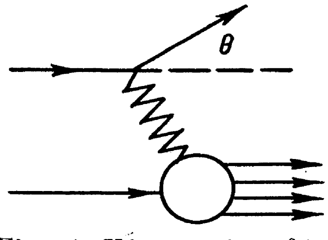

Fig. 1. Kinematics of inelastic electron-proton scattering.

The structure functions are connected with the electromagnetic current commutator

$$W_{\mu\nu}(p, x) = \langle p \,|\, [J_\mu(x), J_\nu(0)] \,|\, p\rangle^c \tag{3}$$

as follows

$$W_{\mu\nu}(p, q) = \frac{1}{4\pi} \int d^4x e^{iqx} W_{\mu\nu}(p, x) =$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1 + \left(p_\mu - \frac{\nu}{q^2} q_\mu\right)\left(p_\nu - \frac{\nu}{q^2} q_\nu\right) W_2. \tag{4}$$

We now formulate the automodelity principle [5]. Let us assume that in describing electromagnetic (or weak) interactions for large energies and momentum transfers none of the dimensional quantities, like masses, «elementary length» etc. are predominant and, thus, the structure functions depend only upon variable invariants. Therefore, when the scale of measurement of the momenta changes by a factor of λ the structure functions of deep inelastic electromagnetic and weak processes are expected to transform as homogeneous functions of appropriate dimensionality.

We apply this principle to electroproduction processes. It is easy to calculate the dimensionality of the tensor and the structure functions *

$$[W_{\mu\nu}(p, q)] = 1, \quad [W_1] = 1, \quad [W_2] = [m^{-2}]. \tag{5}$$

Under scale transformations $p \to \lambda p$, $q \to \lambda q$ from the automodelity principle it follows that

$$W_{\mu\nu}(\lambda p, \lambda q) = W_{\mu\nu}(p, q),\ W_1(\lambda^2 q^2, \lambda^2 \nu) = W_1(q^2, \nu),$$

$$W_2(\lambda^2 q^2, \lambda^2 \nu) = \lambda^{-2} W_2(q^2, \nu). \tag{6}$$

These conditions can be satisfied if we put

$$W_1(\nu, q^2) = F_1\left(\frac{\nu}{q^2}\right)$$

$$\nu W_2(\nu, q^2) = F_2\left(\frac{\nu}{q^2}\right). \tag{7}$$

Thus, although W_1 and νW_2 depend, generally speaking, upon two variables, at large q^2 and ν, according to the automodelity principle, they may become functions of only one dimensionless variable. (In practice it is convenient to use the dimensionless variables ω or x defined according to $\omega = \frac{2\nu}{-q^2} = \frac{1}{x}$. Then in the physical region of electroproduction $1 < \omega < \infty$ and $0 < x < 1$.) Such a behaviour of the structure functions of electroproduction has been predicted by Bjorken [6] on the basis of the connection of the structure functions W_1 and W_2 with

* In the system of units, in which action and velocity are dimensionless the current dimensionality is $[I_\mu] = [m^3]$ and the n-particle state vector a relativistically invariant normalization has the dimensionality $[|p_1, p_2, \ldots, p_n\rangle] = [m^{-n}]$.

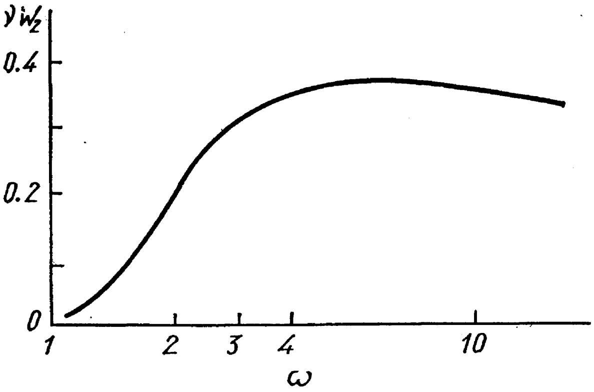

Fig. 2. νW_2 as a function of $\omega = -\frac{2\nu}{q^2}$ from the data at 10^0, and under the assumption $R = \frac{\sigma_L}{\sigma_T} = 0$.

almost equal-time commutators in the limit $q_0 \to i\infty$ and $p_z \to \infty$. This prediction is in rather good agreement with the data at 6°, 10° and 18° (Fig. 2 and for more details see ref. [1]). Thus, it is seen that the data for different q^2 and ν are described by a single universal curve of nontrivial form.

Now apply the automodelity principle to the process $e^+ + e^- \to \ldots$ of annihilation of an electron-positron pair to hadrons. The total cross section of the reaction is described by a single spectral function $\rho(q^2)$ depending upon a single variable q^2 being the square of the energy in the c.m. system

$$\sigma_{\text{tot}}^{e^+e^-}(q^2) = \frac{8\pi^2\alpha^2}{q^2}\rho(q^2). \tag{8}$$

Since $\rho(q^2)$ is dimensionless, according to the automodelity principle $\rho(\lambda^2 q^2) = \rho(q^2) = C$, where C is a constant. If this constant is not zero then the annihilation cross section must behave asymptotically as $\sigma_{\text{tot}}^{e^+e^-} \sim \frac{\text{const}}{q^2}$ analogously to the «point» process $e^+ + e^- \to \mu^+ + \mu^-$. In the x-space this prediction leads to vacuum expectation value of the electromagnetic current commutator being equal to

$$\langle 0 | [J_\mu(x), J_\nu(0)] | 0 \rangle = \frac{ic}{\pi}(g_{\mu\nu}\Box - \partial_\mu\partial_\nu)\,\delta(\vec{x})\,\mathcal{P}\left(\frac{1}{x_0}\right) \tag{9}$$

where $\mathcal{P}$ is the symbol of the principal value. Hence it follows that the vacuum expectation value of the equal-time commutator bet between the time and space components $\langle 0 | [J_0(\vec{x}, 0), J_i(0)] | 0 \rangle$ is equal to the Schwinger term with quadratically divergent c-number coefficient $\lim\limits_{\tau\to 0}\frac{1}{\tau^2}\frac{ic}{\pi}\nabla_i\,\delta(\vec{x})$.

At the next application of the automodelity principle we consider the process of production of a muon pair in deep inelastic collision of two hadrons [7—9]

$$p + p' \to \mu^+ + \mu^- + \cdots \tag{10}$$

described by the diagram of Fig. 3.

The cross section for the process in which in the final state a muon pair alone is detected is expressed in terms of the following second rank tensor

$$\rho_{\mu\nu}(p, p', q) = \sum_N (2\pi)^4\,\delta(p + p' - q - p_N)\,\langle p, p'_{\text{in}} | J_\mu^{\text{em}}(0) | N_{\text{out}}\rangle^c \times$$
$$\times \langle N_{\text{out}} | J_\nu^{\text{em}}(0) | p, p'_{\text{in}}\rangle^c. \tag{11}$$

Using the polarization vectors of the virtual photon we obtain

$$\varepsilon_\mu^{(T_1)} = \frac{1}{\sqrt{-\left(P'^2 - \frac{(P\cdot P')^2}{P^2}\right)}}\left(P'_\mu - \frac{PP'}{P^2}P_\mu\right) \tag{12a}$$

$$\varepsilon_\mu^{(T_2)} = \frac{1}{\sqrt{q^2(pp')^2 - q^2m^2m'^2}}\,\varepsilon_{\mu\alpha\beta\gamma}p^\alpha p'^\beta q^\gamma \tag{12b}$$

$$\varepsilon_\mu^{(L)} = \frac{1}{\sqrt{-P^2}}P_\mu \tag{12c}$$

where

$$P_\mu = p_\mu - \frac{pq}{q^2}q_\mu,\; P'_\mu = p'_\mu - \frac{p'q}{q^2}q_\mu. \tag{13}$$

This tensor may be expanded in five independent structures:

$$\rho_{\mu\nu} = \rho_{T_1}\varepsilon_\mu^{(T_1)}\varepsilon_\nu^{(T_1)} + \rho_{T_2}\varepsilon_\mu^{(T_2)}\varepsilon_\nu^{(T_2)} + \rho_L\varepsilon_\mu^{(L)}\varepsilon_\nu^{(L)} +$$
$$+ \rho_{TL}^{(+)}(\varepsilon_\mu^{(T_1)}\varepsilon_\nu^{(L)} + \varepsilon_\nu^{(T_1)}\varepsilon_\mu^{(L)}) + i\rho_{TL}^{(-)}(\varepsilon_\mu^{(T_1)}\varepsilon_\nu^{(L)} - \varepsilon_\nu^{(T_1)}\varepsilon_\mu^{(L)}). \quad (14)$$

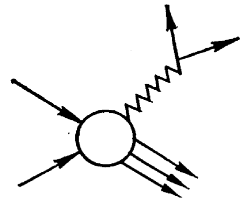

Fig. 3. Kinematics of the $\mu^+\mu^-$ pair production process.

The structure functions or the form factors ρ_i ($i = T_{1,2}$, L, $TL^{(\pm)}$) are real functions depending upon four independent Lorentz-invariant variables which may be chosen as follows ($s = (p + p')^2$, q^2, $\nu = p \cdot q$, $\Delta^2 = (p' - q)^2$. Note that in the c. m. s. of the lepton pair $\vec{q} = 0$ there is a simple relationship between the space components of the tensor and the form factors

$$\begin{pmatrix} \rho_{xx} & 0 & \rho_{xz} \\ 0 & \rho_{yy} & 0 \\ \rho_{zx} & 0 & \rho_{zz} \end{pmatrix} = \begin{pmatrix} \rho_{T_1} & 0 & \rho_{TL}^{(+)} + i\rho_{TL}^{(-)} \\ 0 & \rho_{T_2} & 0 \\ \rho_{TL}^{(+)} - i\rho_{TL}^{(-)} & 0 & \rho_L \end{pmatrix}. \quad (15)$$

The angular distribution of the produced muons is expressed as

$$W(\theta) = \frac{1}{2\rho\left(1 - \frac{V^2}{3}\right)}\left[(\rho_{T_1} + \rho_{T_2})\left(1 - \frac{V^2}{2}\sin^2\theta\right) + \rho_L(1 - V^2\cos^2\theta)\right], \quad (16)$$

where $V = \sqrt{\frac{q^2 - 4m_\mu^2}{q^2}}$ is the muon velocity in their c. m. s. According to the automodelity principle the following asymptotic behaviour of the form factors may be expected for large invariants

$$\rho_i(s, q^2, \nu, \Delta) = F_i(\alpha, \beta, \omega), \qquad i = T_1, T_2, L, TL^{(\pm)} \quad (17)$$

where F_i are the dimensionless functions of the three dimensionless variables α, β and ω. In combination with the current algebra the automodelity principle makes it possible to obtain a number of sum rules for these functions (see below, sect. 2).

In ref. [10] one has studied the process of electroproduction with one hadron distinguished in the final state

$$e^- + p \to e^- + p' + \cdots \quad (18)$$

which proceeds in the one-photon approximation, according to the diagram of Fig. 4.

Owing to crossing symmetry this reaction is closely connected to the process (10) of lepton-antilepton pair production. Some preliminary data on the experimental study of this process (18) are presented at the present Conference [11].

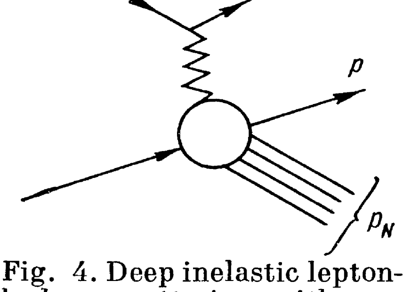

Fig. 4. Deep inelastic lepton-hadron scattering with one hadron distinguished in the final state.

The lepton-antilepton pair production process (10) is tightly connected with a possible process of production of a W-meson or $\mu^+\nu_\mu$ and $\mu^-\bar{\nu}_\mu$ leptons in two-hadron collision

$$p + p' \to \mu^+ + \nu_\mu + \cdots \quad (19)$$

$$p + p' \to \mu^- + \bar{\nu}_\mu + \cdots \quad (20)$$

If the colliding particles are unpolarized and in the final state a muon alone is detected, the cross section, the angular distribution and the muon polarization in processes (19) and (20) may be expressed through the following Hermitian second rank tensors

$$\rho_{\mu\nu}(p, p', q) = \sum_N (2\pi)^4 \delta(p + p' - q - p_N) \langle p, p_{\rm in} | J_\mu^{W}(0) | N_{\rm out}\rangle^c \times \\ \times \langle N_{\rm out} | J_\nu^{W+}(0) | p, p'_{\rm in}\rangle^c \tag{21}$$

$$\bar\rho_{\mu\nu}(p, p', q) = \sum_N (2\pi)^4 \delta(p + p' - q - p_N) \langle p, p_{\rm in} | J_\mu^{W+}(0) | N_{\rm out}\rangle^c \times \\ \times \langle N_{\rm out} | J_\nu^{W}(0) | p, p'_{\rm in}\rangle^c. \tag{22}$$

It is convenient to expand these tensors in the structures corresponding to possible weak current polarizations. Contrary to the virtual photon the weak current may have scalar polarization due to axial current nonconservation. The corresponding polarization four-vector is

$$\varepsilon_\mu^{(S)} = \frac{q_\mu}{\sqrt{q^2}} \tag{23}$$

and the vectors of transverse and longitudinal polarization are as before determined by eqs. (12a) — (12b) *.

It is possible to form from four polarization vectors 16 linearly independent second rank tensors $\varepsilon_\mu^{(\alpha)}\varepsilon_\nu^{(\beta)} \alpha, \beta = S, T_1, T_2, L$. We denote the symmetrized and antisymmetrized combinations of these tensors by $\{\varepsilon_\mu^{(\alpha)}, \varepsilon_\nu^{(\beta)}\} \equiv \varepsilon_\mu^{(\alpha)}\varepsilon_\nu^{(\beta)} + \varepsilon_\nu^{(\alpha)}\varepsilon_\mu^{(\beta)}$ and $[\varepsilon_\mu^{(\alpha)}, \varepsilon_\nu^{(\beta)}] = \varepsilon_\mu^{(\alpha)}\varepsilon_\nu^{(\beta)} - \varepsilon_\nu^{(\alpha)}\varepsilon_\mu^{(\beta)}$ respectively and expand the tensor $\rho_{\mu\nu}$ in 16 independent structures

$$\begin{aligned}\rho_{\mu\nu}(p, p', q) = {} & \rho_S \varepsilon_\mu^{(S)}\varepsilon_\nu^{(S)} + \rho_{T_1}\varepsilon_\mu^{(T_1)}\varepsilon_\nu^{(T_1)} + \rho_{T_2}\varepsilon_\mu^{(T_2)}\varepsilon_\nu^{(T_2)} + \rho_L \varepsilon_\mu^{(L)}\varepsilon_\nu^{(L)} + \\ & + \rho_{ST_1}^{(+)}\{\varepsilon_\mu^{(S)}, \varepsilon_\nu^{(T_1)}\} + i\rho_{ST}^{(-)}[\varepsilon_\mu^{(S)}, \varepsilon_\nu^{(T_1)}] + \rho_{ST_2}^{(+)}\{\varepsilon_\mu^{(S)}, \varepsilon_\nu^{(T_2)}\} + i\rho_{ST_2}^{(-)}[\varepsilon_\mu^{(S)}, \varepsilon_\nu^{(T_2)}] + \\ & + \rho_{SL}^{(+)}\{\varepsilon_\mu^{(S)}, \varepsilon_\nu^{(L)}\} + i\rho_{SL}^{(-)}[\varepsilon_\mu^{(S)}, \varepsilon_\nu^{(L)}] + \rho_{T_1T_2}^{(+)}\{\varepsilon_\mu^{(T_1)}, \varepsilon_\nu^{(T_2)}\} + i\rho_{T_1T_2}^{(-)}[\varepsilon_\mu^{(T_1)}, \varepsilon_\nu^{(T_2)}] + \\ & + \rho_{T_1L}^{(+)}\{\varepsilon_\mu^{(T_1)}, \varepsilon_\nu^{(L)}\} + i\rho_{T_1L}^{(-)}[\varepsilon_\mu^{(T_1)}, \varepsilon_\nu^{(L)}] + \rho_{T_2L}^{(+)}\{\varepsilon_\mu^{(T_2)}, \varepsilon_\nu^{(L)}\} + i\rho_{T_2L}^{(-)}[\varepsilon_\mu^{(T_2)}, \varepsilon_\nu^{(L)}].\end{aligned} \tag{24}$$

The tensor $\rho_{\mu\nu}$ is the density matrix of a spin 1 particle having no definite parity and related to non-conserved current. In the system $\vec q = 0$ there is a simple relationship between the form factors and the components of the tensor $\rho_{\mu\nu}$, i. e. the elements of the density matrix in the linear basis

$$\begin{pmatrix}\rho_{00} & \rho_{0x} & \rho_{0y} & \rho_{0z} \\ \rho_{x0} & \rho_{xx} & \rho_{xy} & \rho_{xz} \\ \rho_{y0} & \rho_{yx} & \rho_{yy} & \rho_{yz} \\ \rho_{z0} & \rho_{zx} & \rho_{zy} & \rho_{zz}\end{pmatrix} = \begin{pmatrix}\rho_S & \rho_{ST_1}^{(+)} + i\rho_{ST_1}^{(-)} & \rho_{ST_2}^{(+)} + i\rho_{ST_2}^{(-)} & \rho_{SL}^{(+)} + i\rho_{SL}^{(-)} \\ \rho_{ST_1}^{(+)} - i\rho_{ST_1}^{(-)} & \rho_{T_1} & \rho_{T_1T_2}^{(+)} + i\rho_{T_1T_2}^{(-)} & \rho_{T_1L}^{(+)} + i\rho_{T_1L}^{(-)} \\ \rho_{ST_2}^{(+)} - i\rho_{ST_2}^{(-)} & \rho_{T_1T_2}^{(+)} - i\rho_{T_1T_2}^{(-)} & \rho_{T_2} & \rho_{T_2L}^{(+)} + i\rho_{T_2L}^{(-)} \\ \rho_{SL}^{(+)} - i\rho_{SL}^{(-)} & \rho_{T_1L}^{(+)} - i\rho_{T_1L}^{(-)} & \rho_{T_2L}^{(+)} - i\rho_{T_2L}^{(-)} & \rho_L\end{pmatrix} \tag{25}$$

In principle, these form factors can be determined from the angular distribution and muon polarization measurements. The contribution to the total cross section comes only from the sum of the diagonal form factors $\rho_S + \rho_{T_1} + \rho_{T_2} + \rho_L$. The check of the automodelity principle predictions for these form factors is difficult. At present we have the estimate for the cross section of production of a W-meson in proton-proton collisions at $P_{\rm lab} = 28.5$ GeV/c obtained in ref. [12].

$$B\sigma_W \leqslant 6 \cdot 10^{-36} cm^2$$

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* It is easily seen that the polarization vectors determined in such a way obey the condition of orthonormality $\varepsilon_\mu^{(\alpha)}\varepsilon^{(\beta)\mu} = g_{\alpha\beta}$, $\alpha,\beta = S, T_{1,2}, L$ and completeness $\varepsilon_\mu^{(S)}\varepsilon_\nu^{(S)} - \sum_{i=T_{1,2},L} \varepsilon_\mu^{(i)}\varepsilon_\nu^{(i)} = g_{\mu\nu}$.

The automodelity principle may also be used in analysing the behaviour of other electromagnetic and weak processes. We note that the automodelity or «self-similar» character of the form factors makes it possible, in the asymptotic domain, firstly, to decrease the number of independent variables by unity and, secondly, knowing the form factors for a given set of invariants, predict their value for another one, provided that their ratios remain fixed.

To our mind, of greatest interest would be the experimental confirmation of the form factor behaviour predicted by the scale invariance principle up to certain large values of the invariants. A deviation from these predictions would mean that a dimensional factor such as «elementary length» or some other which violates automodelity at supersmall distances is coming in play. The problem of automodelity or scale invariance violation appears to be tightly associated with conformal symmetry violation up to the Poincare group. The currents of scale and special conformal transformations $S^{\mu} = \theta^{\mu\alpha}X_{\alpha}$ and $C^{\mu\nu} = \theta^{\mu\alpha}(2X^{\nu}X_{\alpha} - g_{\alpha}^{\nu}x^2)$ in a wide class of the Lagrange field theories conserve if the trace of the energy-momentum tensor is zero. When in the Lagrangian the dependence upon masses and other dimensional constants is absent $\theta_{\mu}^{\mu} = 0$, which leads not only to scale, but also to complete conformal symmetry.

2. Current commutators and asymptotic sum rules

The combination of the automodelity principle with the current algebra makes it possible to derive a number of sum rules capable of giving some information on the structure of the electromagnetic and weak hadron currents.

For definiteness, we consider the derivation of the sum rules for the process of muon pair production in strong interactions [7].

The Fourier transform of the diagonal matrix element of the electromagnetic current commutator

$$R_{\mu\nu}(p, p', q) = \int d^4x e^{-iqx} \langle p, p'_{\text{in}} | [J_{\mu}^{\text{e.m.}}(x), J_{\nu}^{\text{e.m.}}(0)] | p, p'_{\text{in}}\rangle_c \tag{26}$$

is connected with the quantity $\rho_{\mu\nu}(p, p', q)$ defining the cross section for process (10) by the relation

$$R_{\mu\nu}(p, p', q) = \rho_{\mu\nu}(p, p', q) + \tilde{\rho}_{\mu\nu}(p, p', q) - \rho_{\nu\mu}(p, p', -q) - \tilde{\rho}_{\nu\mu}(p, p', -q) \tag{27}$$

where $\tilde{\rho}_{\mu\nu}$ denotes the contribution of 15 weakly coupled z-diagrams. From the four momentum conservation and spectrality it follows that for $q^2 > 0$:

$$\rho_{\mu\nu}(p, p', q) = \theta(\nu)\,\theta((\sqrt{s} - \sqrt{q^2})^2 - m_N^2)\,\rho_{\mu\nu}(p, p', q) \tag{28}$$

$$\tilde{\rho}_{\mu\nu}(p, p', q) = \theta(-\nu)\,\theta(m_N^2 - (\sqrt{s} + \sqrt{q^2})^2)\,\tilde{\rho}_{\mu\nu}(p, p', q) \tag{29}$$

and thus, in the physical domain the contribution of $\tilde{\rho}_{\mu\nu}(p, p', q)$ is exactly zero. However, in deducing the sum rules one uses the whole domain, both physical and unphysical ones, therefore the non-zero contribution may be given by the z-diagrams from the second part of the commutator $\tilde{\rho}_{\nu\mu}(p, p' - q)$. It can be shown that under the assumptions commonly accepted in deducing the sum rules by means of the current algebra the contribution of these diagrams tends to zero at $s \to \infty$. As is seen from condition (29) in this limit the contributions of the z-diagrams are defined by the intermediate states of hadrons with infinitely heavy effective masses m_N^2. In fact, the contribution of the diagrams is determined by

$$\frac{1}{\pi}\int_0^\infty \rho_{TL}^{(-)}(\vec{p}, \vec{p}', -|q_0|) = -\frac{1}{\pi}\int_s^\infty \frac{dm_N^2}{2E_N}\rho_{TL}^{(-)}(\vec{p}, \vec{p}' -|q_0|). \tag{30}$$

Assuming that the order of integration and passing to the limit $s \to \infty$ may be changed, and going over to this limit under the integral sign for a fixed m_N^2 we find that the contribution of the diagrams in the sum rules vanishes in this limit.

Finally, these sum rules read

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega F_{TL}^{(-)}(\alpha, \beta, \omega) = B_{xz}(\alpha, \beta) - B_{zx}(\alpha, \beta) \tag{31a}$$

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega\omega F_{T_1}(\alpha, \beta, \beta) = C_{xx}(\alpha, \beta) \tag{31b}$$

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega\omega F_{T_2}(\alpha, \beta, \omega) = C_{yy}(\alpha, \beta) \tag{31c}$$

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega\omega F_{L}(\alpha, \beta, \omega) = C_{zz}(\alpha, \beta) \tag{31d}$$

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega\omega F_{TL}^{(+)}(\alpha, \beta, \omega) = C_{xz}(\alpha, \beta) - C_{zx}(\alpha, \beta) \tag{31e}$$

where α, β and ω are certain dimensionless combinations of invariants (see refs. [7—9]) and the dimensionless quantities $B_{ij}(\alpha, \beta)$ and $C_{ij}(\alpha, \beta)$ are defined as

$$B_{ij}(\alpha, \beta) = \lim_{\substack{p_0, p_0' \to\infty \\ \alpha,\beta\text{—fixed}}} p_0 B_{ij}(\vec{p}, \vec{p}'),\ C_{ij}(\alpha, \beta) = \lim_{\substack{p_0, p_0' \to\infty \\ \alpha,\beta\text{—fixed}}} C_{ij}(\vec{p}, \vec{p}') \tag{32}$$

$$B_{ij}(\vec{p}, \vec{p}') = -i\int d\vec{x}\langle p, p_{\text{in}}' |[J_i^{\text{e.m.}}(\vec{x}, 0), J_j(0)]| p, p_{\text{in}}\rangle^c \tag{33}$$

The right-hand sides of equalities (31) essentially depend on the choice of the model and therefore may serve as a criterion for choosing one or another model defining the structure of the hadron electromagnetic current. In particular, for the polarization form factor it follows from (24a) that

$$\int_0^{\omega_0} d\omega F_{TL}^{(-)}(\alpha, \beta, \omega) = \begin{cases}\text{const—quark model} \\ 0\text{—field algebra.}\end{cases} \tag{34}$$

3. Vector dominance

Reliable information on the *di*-muon mass spectrum may be obtained using the vector meson dominance hypothesis [8, 9, 13]. To this end it is convenient to represent the exact formula for the mass spectrum in the form:

$$\frac{d\sigma}{dq^2} = \frac{\alpha}{2\pi}\frac{1}{q^2}\left(1 - \frac{q^2 - 4m_\mu^2}{3q^2}\right)\sqrt{\frac{q^2 - 4m_\mu^2}{q^2}}\,\sigma^{\gamma^*}(s, q^2) \tag{35}$$

where σ^{γ^*} is the total cross section for the virtual γ^* photon production in the process $p + p' \to \gamma^* + \cdots$. According to the vector dominance hypothesis it relates to the total cross section for the real vector meson production in the process $p + p' \to V + \cdots$ as follows:

$$\sigma^{\gamma^*}(s, q^2) = \frac{\alpha}{4}\left[\left(\frac{m_\rho^2}{m_\rho^2 - q^2}\right)^2 \frac{4\pi}{\gamma_\rho^2}\sigma^\rho(s) + \left(\frac{m_\omega^2}{m_\omega^2 - q^2}\right)^2 \frac{4\pi}{\gamma_\omega^2}\sigma^\omega(s) + \right.$$

$$\left. + \left(\frac{m_\Phi^2}{m_\Phi^2 - q^2}\right)^2 \frac{4\pi}{\gamma_\Phi^2}\sigma^\Phi(s)\right] + \text{interference terms.} \tag{36}$$

Inserting this approximate value for σ^{γ^*} in (35), neglecting the muon mass assuming the interference term contribution to be small we get the following expression for the muon pair mass spectrum:

$$\frac{d\sigma}{dq^2} = \frac{\alpha^2}{12\pi}\sum_{V=\rho^0,\omega,\Phi}\left(\frac{m_V^2}{m_V^2 - q^2}\right)^2 \frac{4\pi}{\gamma_V^2}\sigma^V(s). \tag{37}$$

It is known that the Φ-meson is weakly produced in nonstrange hadron collisions. Therefore, keeping only the contribution of the ρ^0 and ω mesons and assuming $m_\rho \simeq m_\omega$, $\gamma_\rho^2 : \gamma_\omega^2 = 1 : 9$, $\frac{\gamma_\rho^2}{4\pi} = 0.5$ we reduce (37) to the form ($m_{\mu\mu} = \sqrt{q^2}$):

$$\frac{d\sigma}{dm_{\mu\mu}} = \frac{2 \cdot 10^{-6}}{m_{\mu\mu}(m_{\mu\mu}^2 - 0.6)^2}\left[\sigma^\rho(s) + \frac{1}{9}\sigma^\omega(s)\right]\frac{cm^2}{GeV} \tag{38}$$

or, for large

$$\frac{d\sigma}{dm_{\mu\mu}} = \frac{2 \cdot 10^{-6}}{m_{\mu\mu}^5}\left[\sigma^\rho(s) + \frac{1}{9}\sigma^\omega(s)\right]\frac{cm^2}{GeV}. \tag{39}$$

We apply (38) to the analysis of the muon pair production process for some hadron — hadron collisions.

A. Proton-proton collisions.

The production of the ρ^0-meson in the reaction $p + p \to p + p + \rho^0$ was not observed over the whole interval up to $p_{\text{lab}} = 28.5\ GeV/c$. In the same interval the ω-meson production cross sections in the reaction $p + p \to p + p + \omega$ are

p_{lab}	$5\ \frac{GeV}{c}$	$10\ \frac{GeV}{c}$	$28.5\ \frac{GeV}{c}$
σ^ω	$140 \pm 20\mu b$	$60\mu b$	$50 \pm 10\mu b$

This fact is in agreement with the analysis based on the double Regge-pole model. The analysis of the six-prong reaction

$$p + p \to p + p + \pi^+\pi^+\pi^-\pi^-$$

shows that about 24% of the events proceed through the production of a ρ^0-meson which corresponds to the cross section 90 μb. Assuming that in this case too, about $^1/_4$ of the events proceed through the production of ρ^0 the corresponding cross section is estimated to be about 5 μb. Thus, the total cross section for ρ^0 production in pp collisions for $p_{\text{lab}} = 28.5\ GeV/c$ may be assumed to be about 100 μb

$$\sigma^{pp\to\rho^0+\cdots} \gtrsim 100\mu b. \tag{40}$$

If it is supposed that $\sigma^\omega \approx \sigma^\rho \approx 100\ \mu b$, then for the mass spectrum of a *di*-muon produced in pp collisions with $p_{\text{lab}} = 28.5\ GeV/c$ we finally get from (38) or (39) the following expression

$$\frac{d\sigma}{dm_{\mu\mu}} = \frac{2.2 \cdot 10^{-34}}{m_{\mu\mu}(m_{\mu\mu}^2 - 0.6)^2}\cdot\frac{cm^2}{GeV} \approx \frac{2.2 \cdot 10^{-34}}{m_{\mu\mu}^5}\cdot\frac{cm^2}{GeV}. \tag{41}$$

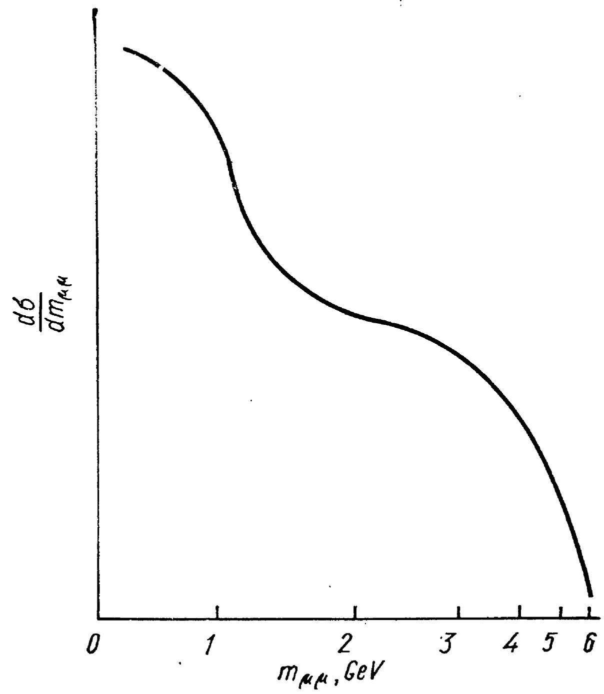

Fig. 5. Prediction of the vector dominance model for the mass spectrum of the di-muon produced in proton — proton collision with $p_{\text{lab}} = 28.5\ GeV/c$ according formula (41).

This formula qualitatively describes well the observed spectrum up to $m_{\mu\mu} \approx \approx 4.5\ GeV$. Fig. 5 shows the experimental spectrum measured in the work of the Columbia — BNL group [3]. The steep fall off of the spectrum above 4.5 *GeV* is due to the reduction of the phase space.

B. Pionproton collisions.

We consider π^+p collision. On the basis of the analysis performed in ref. [13] may be concluded that the cross section for production of a ρ^0-meson in the process $\pi^+ + p \to \rho^0 + \cdots$ is larger or nearly equal to 1840 μb for $p = 8.5\ GeV/c$.

$$\sigma^{\pi^+p\to\rho^0+\cdots} \geqslant 1840\,\mu b \qquad (42)$$

and the cross section for ω production in the process $\pi^+p \to \omega + \cdots$

$$\sigma^{\pi^+p\to\omega+\cdots} \geqslant 200\,\mu b. \qquad (43)$$

From the above considerations and eqs. (38) and (39) we get the following approximate lower estimate for the mass spectrum of the *di*-muon produced in π^+p-collisions with lab. momentum 8.5 GeV/c

$$\frac{d\sigma^{\pi^+p}}{dm_{\mu\mu}} = \frac{3.7\cdot 10^{-33}}{m_{\mu\mu}(m^2_{\mu\mu}-0.6)^2}\cdot\frac{cm^2}{GeV} = \frac{3.7\cdot 10^{-33}}{m^5_{\mu\mu}}\cdot\frac{cm^2}{GeV}. \qquad (44)$$

In this short review we have mainly considered the work of the Dubna group. Other theoretical papers, concerning the parton model, light cone singularities and operator product expansion are discussed in L. D. Soloviev's rapporteur talk [14] where the appropriate references can be found.

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